

hep-th/9704004

IFUM 527/FT

April, 97

# Infrared Stability of $N = 4$ Super Yang-Mills Theory<sup>1</sup>

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## Abstract

We study the infrared perturbative properties of a class of non supersymmetric gauge theories with the same field content of  $N = 4$  Super Yang-Mills and we show that the  $N = 4$  supersymmetric model represents an IR unstable fixed point for the renormalization group equations.

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<sup>1</sup>Partially supported by EEC, Science Project ERBFMRX-ct96-0045.

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# 1 Introduction.

In this letter we analyse the infrared stability of  $N = 4$  supersymmetric gauge theories. We consider the embedding of the supersymmetric theory in a space of more general models which have the same field content and gauge group as the supersymmetric one, but do not possess any supersymmetry. The symmetric theory is a fixed point of the renormalization group equations. We discuss the infrared stability of such a solution.

It has been a long standing idea that the renormalization group flow could lead to a more symmetric phase in the infrared (or in the ultraviolet). This idea has been applied to several models [1, 2, 3, 4, 5, 6], among which supersymmetric gauge model [7, 8]. In cases where the models considered have a residual  $N = 1$  supersymmetry more powerful methods exist which rely on the holomorphicity properties of the models and the structure of the exact beta functions [9, 10]. This approach is also related to problems like the analysis of the stability of BRS gauge invariant theory [11] or the treatment of QCD with boundaries [12]. Moreover, there are recent examples [13] of gauge theories in different dimensions which are conjectured to flow in the IR to non-trivial fixed points with enhanced global symmetries.

In this letter we limit ourselves to the perturbative analysis of the RG flow in a class of gauge theories which represents a neighborhood of  $N = 4$  supersymmetric gauge theory. All these models admit a lagrangian description and we consider only renormalizable theories (for an analysis of infrared stability of supersymmetric theories in terms of non renormalizable effective lagrangians, see [14]).

We concentrate our attention on the stability of  $N = 4$  super Yang-Mills with gauge group  $SU(2)$ . This model has already been studied [8] in the case of a residual  $N = 1$  supersymmetry and turned out to be IR stable, while for groups others than the so called safe algebras the same model gave an unstable  $N = 4$  fixed point. We show that the stability in the case of  $SU(2)$  can be ascribed to the residual supersymmetry of the model. We find indeed that if we do not impose an  $N = 1$  residual supersymmetry, the  $N = 4$  supersymmetric theory becomes an IR unstable fixed point even in the case of safe algebras.

In section 2 we briefly review the results for the  $N = 1$  supersymmetric case. In section 3 we extend this analysis to the case of a general non supersymmetric model and we show that the  $N = 4$  fixed point is unstable.

## 2 $N = 1$ supersymmetric case.

$N = 4$  supersymmetric gauge theories can be thought of as  $N = 1$  gauge theories with three chiral superfields in the adjoint representation of the gauge group  $G$ ,  $\Phi^i = \sum_a \Phi_a^i T^a$  ( $i = 1, 2, 3; a = 1, \dots, \dim G$ ), coupled through the superpotential  $\lambda \varepsilon_{ijk} \text{Tr}(\Phi^i [\Phi^j, \Phi^k])$ . The group generators  $T_a$  are in the fundamental representation.  $N = 4$  supersymmetry forces the coupling  $\lambda$  of the chiral superpotential to be equal to the gauge coupling  $g$ .

One can relax the  $N = 4$  constraint and consider a more general family of models: they have the same field content as  $N = 4$  Super Yang-Mills (three chiral superfields and one vector superfield), are invariant under the gauge group  $G$  and still realize only one of the four supersymmetries.

The generic theory belonging to this family is described by the following Lagrangian

$$\begin{aligned} \mathcal{L} = & \int d^4\theta \sum_{i=1}^3 \text{Tr}(e^{-gV} \bar{\Phi}_i e^{gV} \Phi^i) + \frac{1}{64g^2} \int d^2\theta \text{Tr}(W^\alpha W_\alpha) + \\ & + \left( \frac{i\lambda}{3!} \int d^2\theta \varepsilon_{ijk} \text{Tr}(\Phi^i [\Phi^j, \Phi^k]) + \frac{f_{ijk}}{3!} \int d^2\theta \text{Tr}(\Phi^i \{\Phi^j, \Phi^k\}) \right) + h.c.) + \\ & + \mathcal{L}_{G.F} + \mathcal{L}_{F.P} \end{aligned} \quad (1)$$

where  $f_{ijk}$  is a totally symmetric constant tensor,  $W_\alpha = \bar{D}^2(e^{-gV} D_\alpha e^{gV})$  is the gauge field strength, and  $\mathcal{L}_{G.F}, \mathcal{L}_{F.P}$  are the usual gauge fixing and ghost superfield Lagrangians [15, 16].

The two terms  $\text{Tr}(\Phi^i [\Phi^j, \Phi^k])$  and  $\text{Tr}(\Phi^i \{\Phi^j, \Phi^k\})$  are the most general renormalizable interactions compatible with  $N = 1$  supersymmetry.

The former term is the usual  $N = 4$  potential for the scalar superfields, while the latter, forbidden in extended supersymmetry, is non zero only for  $G = SU(N \geq 3)$  or  $E_8$ , i.e. for groups that admit a totally symmetric invariant tensor.

$N = 4$  supersymmetry [17] is recovered for the following values of the parameters

$$\lambda = g \quad f_{ijk} = 0 \quad \forall i, j, k$$

and represents a line of fixed point of the renormalization group equations, which, at one loop level, read ( $a$  and  $b$  are positive constants):

$$\beta_g = 0$$

$$\begin{aligned}\beta_\lambda &= a\lambda(\lambda^2 - g^2) \\ \beta_{f_{ijk}} &= bf_{ijk}(\lambda^2 - g^2).\end{aligned}\tag{2}$$

The behaviour of the beta functions in the parameter space around the line of fixed points shows that their stability relies on the presence of the coupling  $f_{ijk}$  and is therefore gauge group dependent: for safe algebras [18], which do not allow for the  $f_{ijk}$  coupling,  $N = 4$  theory is an infrared attractor, while it is not an attractor, neither infrared nor ultraviolet, for other groups ( $SU(N)$  or  $E_8$ ). Similar results can also be obtained, without any recourse to explicit one loop computation, from simple considerations on the form of the beta functions and the properties of  $N = 1$  supersymmetry [10].

### 3 Nonsupersymmetric case.

It is likely that the infrared stability of the fixed points line in the case of safe algebras is simply a consequence of the  $N = 1$  residual supersymmetry. This is indeed suggested by a similar analysis carried out in [7] on a class of gauge theories with the same field content as  $N = 2$  Super Yang-Mills, which shows that the supersymmetric theory is an infrared unstable fixed point.

In the case of interest here, i.e.  $SU(2)$  gauge group,  $N = 1$  supersymmetry forces the parameter space to be two dimensional: the gauge and the Yukawa coupling constants.

Without any supersymmetry constraint there are many more interactions allowed among the fields of the model. It is possible that some of these new couplings turns out to be relevant in the neighborhood of the fixed point, introducing instability directions.

We consider the simplest renormalizable model one can build out of the same fields as  $N = 4$  super Yang-Mills but without  $N = 1$  supersymmetry. We assume non abelian gauge invariance to be an exact symmetry of our class of models; it could eventually be spontaneously broken at one loop [19, 8, 7].

We choose to work in four component notation, where the  $N = 4$  multiplet is represented by [16]:

- $A_\mu$  is the gauge vector, with field strength  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ie[A_\mu, A_\nu]$ ,
- $\lambda$  is a Majorana fermion which represents the gaugino,

- $\psi_i, (i = 1, 2, 3)$ , are three matter Majorana fermions,
- $A_i, (i = 1, 2, 3)$ , are three matter scalars,
- $B_i$  ( $i = 1, 2, 3$ ) are three matter pseudoscalars.

All fields are in the adjoint representation of  $SU(2)$ . With this choice of the fields, the on shell Lagrangian for this models is

$$\begin{aligned}
\mathcal{L} = & Tr \left\{ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{i}{2} \bar{\lambda} D \lambda + \frac{1}{2} (D_\mu A_i)^2 + \frac{1}{2} (D_\mu B_i)^2 - ie(\bar{\psi}_i[\lambda, A_i] + \bar{\psi}_i \gamma_5[\lambda, B_i]) + \right. \\
& - \frac{i\lambda}{2} \varepsilon_{ijk} (\bar{\psi}_i[\psi_j, A_k] - \bar{\psi}_i \gamma_5[\psi_j, B_k]) \left. \right\} - f_1 [Tr(A_i A_i + B_i B_i)]^2 + \\
& + 2(f_1 - f_2) [Tr(A_i A_i) Tr(B_i B_i) - (Tr(A_i B_i))^2] + f_3 [Tr(A_i A_j + B_i B_j)]^2 + \\
& + 2(f_5 - f_3) [Tr(A_i A_j) Tr(B_i B_j) - Tr(A_i B_j) Tr(A_j B_i)] + \\
& + 2f_4 [(Tr(A_i B_j))^2 - Tr(A_i B_j) Tr(A_j B_i)] + \mathcal{L}_{G.F} + \mathcal{L}_{F.P}
\end{aligned} \tag{3}$$

where the covariant derivative is defined as  $D_\mu = \partial_\mu + ie[A_\mu, \cdot]$  and the gauge fixing and ghost Lagrangians are the usual ones [20].

Radiative corrections and one loop renormalizability require the introduction of the additional gauge invariant interactions between scalars and pseudoscalars. In facts, these are all the possible independent interaction terms renormalizable by power counting that one can construct out of the scalars and the pseudoscalars of the model.

In principle, we could have considered a more general Lagrangian with an independent coupling constant for each interaction term, getting a larger parameter space.

The particular choice of the coupling constants of eq. (3) is a consequence of an additional  $U(1)$  symmetry we imposed on the Lagrangian in order to get the simplest possible model which exhibits an unstable behaviour.

This is a non anomalous  $R$ -symmetry of the original  $N = 4$  Lagrangian, which leaves the vector field unchanged and acts on the other fields as

$$\begin{aligned}
A + iB &\rightarrow e^{-i\theta} (A + iB) \\
\psi_i &\rightarrow e^{-\frac{1}{2}\gamma_5\theta} \psi_i \\
\lambda &\rightarrow e^{\frac{3}{2}\gamma_5\theta} \lambda \\
A_\mu &\rightarrow A_\mu.
\end{aligned} \tag{4}$$

The  $U(1)$  symmetry makes the scalars and pseudoscalars interactions symmetrical and so it reduces the parameter space of the theory: the Yukawa terms for  $A$  e  $B$  have the same coupling constant and the four scalars (pseudoscalars) couplings are related to the mixed terms  $A^2 B^2$ .

Lagrangian (3) reduces to the usual  $N = 4$  on-shell Lagrangian [17] when

$$f_5 = 0, g = e = \lambda, f_i = g^2 \quad i = 1, \dots, 4,$$

while we recover the  $N = 1$  models of Antoniadis et al.(eq. (1)) [8] for

$$f_5 = \lambda^2 - g^2, e = \lambda, f_i = \lambda^2 \quad i = 1, \dots, 4.$$

To determine the fixed points of the model, we need to find the zeros of the beta functions for the various couplings. We calculate them at one loop in the minimal subtraction scheme with dimensional regularisation.

At one-loop level the presence of the new quartic terms in the Lagrangian does not affect the propagators and the  $1PI$  functions which enter the computation of the beta-functions for the Yukawa coupling constant  $\lambda$ . So we expect to find the same  $\beta_\lambda$  as in the  $N = 1$  case. Similarly, since the one-loop beta function for the gauge coupling depends only on the field content of the model, which is kept the same as in  $N = 4$ , we reproduce the  $\beta_g = 0$  result of  $N = 4$  case.

In the analysis of the infrared behaviour of this class of models, we are interested in the relative evolution of the various coupling parameters with respect to the scale independent gauge coupling  $g$  ( $\beta_g = 0$ ). To this purpose it is more convenient to define effective couplings ratios [7]:

$$E(t) = \frac{e^2(t)}{g^2(t)}, \quad \Lambda(t) = \frac{\lambda^2(t)}{g^2(t)}, \quad F_i(t) = \frac{f_i(t)}{g^2(t)} \quad i = 1, \dots, 5 \quad (5)$$

In term of these new couplings, the one-loop beta-functions considerably simplify and become:

$$\begin{aligned} \beta_E &= 24hg^2 E(E - 1) \\ \beta_\Lambda &= 24hg^2 \Lambda(\Lambda - 1) \\ \beta_{F_1} &= hg^2(-24F_1 + 16F_1\Lambda + 16EF_1 + 6 - 8\Lambda^2 - 8E^2 - 16E\Lambda + \\ &\quad + 36F_1^2 + 16F_2^2 + 2F_4^2 + 2F_5^2 - 8F_2F_5 + 8F_4F_5 - 28F_1F_3 + \\ &\quad - 4F_3F_4 - 4F_2F_3 + 8F_3^2 - 4F_1F_5 - 4F_3F_5 - 8F_2F_4) \end{aligned}$$

$$\begin{aligned}
\beta_{F_2} &= hg^2(-24F_2 + 16F_2\Lambda + 16EF_2 + 6 - 8\Lambda^2 - 8E^2 - 16E\Lambda + \\
&\quad + 36F_1F_2 + 10F_2^2 - 4F_1F_3 - 28F_2F_3 - 12F_1F_5 + 4F_3F_5 + \\
&\quad + 2F_5^2 + 2F_4^2 - 8F_1F_4 + 4F_3F_4 + 6F_1^2) \\
\beta_{F_3} &= hg^2(-24F_3 + 16F_3\Lambda + 16EF_3 - 6 + 8\Lambda^2 + 8E^2 + \\
&\quad - 16E\Lambda + 28F_1F_3 - 4F_2F_3 + 8F_3F_5 - 20F_3^2 + \\
&\quad + 8F_4F_5 - 2F_5^2 - 2F_4^2 - 4F_3F_4) \\
\beta_{F_4} &= hg^2(-24F_4 + 16F_4\Lambda + 16EF_4 - 6 + 8\Lambda^2 + 8E^2 + \\
&\quad - 16E\Lambda + 8F_1F_3 + 20F_2F_4 - 8F_2F_3 - 10F_3^2 + 12F_3F_5 + \\
&\quad - 2F_5^2 - 16F_4^2 + 4F_1F_4 - 8F_4F_5) \\
\beta_{F_5} &= hg^2(-24F_5 + 16F_5\Lambda + 16EF_5 + 8\Lambda^2 + 8E^2 + \\
&\quad - 16E\Lambda + 8F_1F_3 + 4F_1F_5 - 8F_2F_3 + 20F_2F_5 - 10F_3^2 + \\
&\quad - 16F_5^2 - 2F_4^2 - 8F_4F_5 + 12F_3F_4)
\end{aligned} \tag{6}$$

where  $h = 16\pi^2$ .

One can easily check that the point corresponding to the  $N = 4$  "phase"

$$F_5 = 0, E = \Lambda = F_i = 1 \quad i = 1, \dots, 4,$$

is indeed a solution of system (6), while a numerical analysis of the same system shows the existence of other 25 fixed point solutions in addition to the  $N = 4$  supersymmetric one. They all correspond to ordinary gauge field theories with Yukawa and scalar interactions, without any supersymmetry.

It is therefore excluded the possibility that the class of models we considered could flow to a  $N = 1$  or  $N = 2$  supersymmetric theory. The question is now whether the  $N = 4$  fixed point is stable or not.

The stability properties of a fixed point are determined by the linearization of the renormalization group equations (6), that control the behaviour of the couplings in the neighbourhood of the fixed point. More precisely, the fixed point is stable in the infrared if the matrix representing the linearized system has all positive eigenvalues.

We computed eigenvalues and eigenvectors of the linearized system around each fixed point. For simplicity we list here the main results only.

The  $N = 4$  fixed point represents a saddle point in the parameter space. The linearized system around this point has indeed one negative eigenvalue:

$$\rho_1 = -3 \quad \rho_2 = \rho_3 = \rho_4 = 6 \quad \rho_{5,6} = 3 \pm \sqrt{3} \quad \rho_7 = 16. \tag{7}$$

The corresponding eigenvector represents an instability direction in the parameter space. Moreover we find that, moving to the infrared along this instability direction the system is attracted toward a non supersymmetric fixed point, characterized by the following values of the couplings:

$$E = \Lambda = 1, \quad F_1 = F_2 = 0.757, \quad F_3 = F_4 = 0.352, \quad F_5 = 0, \quad (8)$$

This point turns out to be the unique IR stable fixed point. All others points are neither infrared nor ultraviolet attractors.

Combining this result with the previous ones [7, 8], we draw the conclusion that supersymmetric gauge theory are always unstable in the infrared, though attractive, for every choice of the gauge group.

I would like to thank L. Girardello for suggesting the subject and for useful conversations. I would like to acknowledge helpful discussions with G. Bottazzi, A. Pasquinucci, G. Salam and A. Zaffaroni. This work was supported by the European Commission TMR programme ERBFMRX-CT96-0045, in which M.P. is associated to the University of Torino.

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